- MASTER INTERNSHIP PROPOSAL -

HIGH-ORDER NUMERICAL APPROXIMATION OF ELECTROMAGNETIC SYSTEMS

An electromagnetic system (transformer, synchronous machine, induction machine) is constituted of different media such as electrical iron, copper coil, air, or magnet.

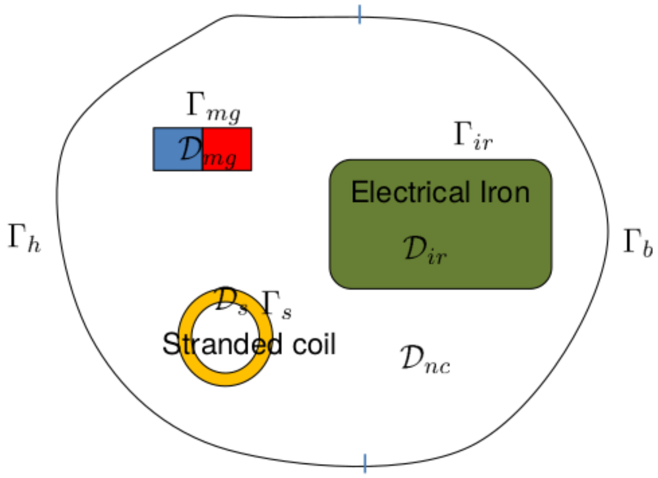

Figure 1: Studied magnetostatic system

At low frequency, if eddy current can be neglected, the system is governed by the equations of magnetostatics:

$$\begin{cases} \mathbf{curl}\boldsymbol{H} = \boldsymbol{J}_s & \text{in } \mathcal{D}_s, \\ \mathrm{div}\boldsymbol{B} = 0 & \text{in } \mathcal{D}_s, \end{cases}$$

$$\begin{cases} \mathbf{curl}\boldsymbol{H} = \mathbf{0} & \text{in } \mathcal{D}_{nc} \cup \mathcal{D}_{ir} \cup \mathcal{D}_{mg}, \\ \mathrm{div}\boldsymbol{B} = 0 & \text{in } \mathcal{D}_{nc} \cup \mathcal{D}_{ir} \cup \mathcal{D}_{mg}, \end{cases}$$

$$\begin{cases} \boldsymbol{B} = \mu(H)\boldsymbol{H} & \text{in } \mathcal{D}_{ir}, \\ \boldsymbol{B} = \mu\boldsymbol{H} & \text{in } \mathcal{D}_{nc}, \\ \boldsymbol{B} = \mu(H)\boldsymbol{H} + \boldsymbol{B}_r & \text{in } \mathcal{D}_{mg}. \end{cases}$$

This model is supplemented with some conditions at the interfaces between the different media. Indeed, the continuity of the tangential component(s) of the magnetic field ($\boldsymbol{H}$) and of the normal component of the magnetic flux ($\boldsymbol{B}$) must be ensured.

$$\begin{cases} \boldsymbol{H}_{ir} \times \boldsymbol{n} = \boldsymbol{H}_{nc} \times \boldsymbol{n} & \text{on } \Gamma_{ir}, \\ \boldsymbol{B}_{ir} \cdot \boldsymbol{n} = \boldsymbol{B}_{nc} \cdot \boldsymbol{n} & \text{on } \Gamma_{ir}, \end{cases}$$

$$\begin{cases} \boldsymbol{H}_{mg} \times \boldsymbol{n} = \boldsymbol{H}_{nc} \times \boldsymbol{n} & \text{on } \Gamma_{mg}, \\ \boldsymbol{B}_{mg} \cdot \boldsymbol{n} = \boldsymbol{B}_{nc} \cdot \boldsymbol{n} & \text{on } \Gamma_{mg}, \end{cases}$$

$$\begin{cases} \boldsymbol{H}_{s} \times \boldsymbol{n} = \boldsymbol{H}_{nc} \times \boldsymbol{n} & \text{on } \Gamma_{s}, \\ \boldsymbol{B}_{s} \cdot \boldsymbol{n} = \boldsymbol{B}_{nc} \cdot \boldsymbol{n} & \text{on } \Gamma_{s}, \\ \boldsymbol{J}_{s} \cdot \boldsymbol{n} = 0 & \text{on } \Gamma_{s}. \end{cases}$$

In 3D it is also possible to use scalar or vector potential formulations. For the moment, the numerical codes at L2EP for the approximation of electromagnetism problems are based on low-order edge finite elements on tetrahedra [2].

In this internship, the objective is to investigate the numerical approximation of such problems by the so-called Hybrid High-Order (HHO) methods. These methods have been recently introduced [1]. They enable an approximation on meshes featuring fairly general element shapes, and naturally allow for an arbitrary order of approximation. The use of general meshes is particularly interesting for adaptive simulations (to simplify the refinement/coarsening procedures), as well as for the coupling between models. The use of a high polynomial degree of approximation is motivated by the fact that, in regions where more accuracy is needed, whenever the solution is smooth enough (locally), increasing the polynomial degree is more efficient in terms of computational cost than refining the mesh.

The HHO technology has been very recently adapted to magnetostatics, and a first method has been devised. This method has been implemented in a 3D `C++` platform [3], and validated on very simple test-cases.

The main task of this internship will be to validate the newly devised method on test-cases relevant for the researchers at L2EP, like the electromagnetic systems described above, with potentially nonlinear constitutive laws. All the implementation will be done in the aforementioned `C++` platform. Depending on time and on the candidate's motivation and abilities, a second task will be to devise, implement, and possibly mathematically analyze variants of the method already developed. In particular, the candidate will focus on potential formulations.

Pre-requisites: A background in the numerical approximation of PDEs, as well as a good knowledge of `C++` programming are mandatory.

Localization: L2EP and Inria Lille - Nord Europe, Villeneuve d'Ascq, France

Duration: 4 to 6 months

Monthly net salary: around 600 euros

Supervision: Yvonnick Le Menach (L2EP) and Simon Lemaire (Inria)

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References

[1] D. A. Di Pietro, A. Ern, and S. Lemaire, *An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators*, Comput. Meth. Appl. Math., 14(4):461–472, 2014.

[2] P. Monk, *Finite element methods for Maxwell's equations*, Numerical Mathematics and Scientific Computation, Oxford University Press, New York, 2003.

[3] M. Cicuttin, D. A. Di Pietro, and A. Ern, *Implementation of Discontinuous Skeletal methods on arbitrary-dimensional, polytopal meshes using generic programming*, J. Comput. Appl. Math., 344:852–874, 2018.